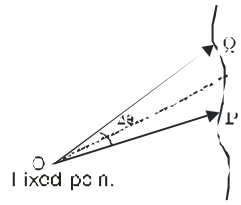
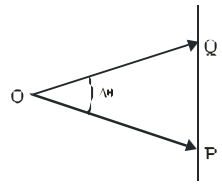
CIRCULAR MOTION

1. ANGULAR DISPLACEMENT

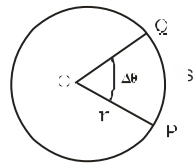
Introduction: Angle subtended by position vector of a particle moving along any arbitrary path w.r.t. some fixed point is called angular displacement.



(a) Particle moving in an arbitrary path



(b) Particle moving in straight line



(c) Particle moving in circular path

(i) Angular displacement is a vector quantity.

(ii) Its direction is perpendicular to plane of rotation and given by right hand screw rule.

Note: Clockwise angular displacement is taken as negative and anticlockwise displacement as positive.

$$\text{angle} = \frac{\text{arc}}{\text{radius}} = \frac{\text{linear displacement}}{\text{radius}}$$

(iii) For circular motion $\Delta S = r \times \Delta\theta$

(iv) Its unit is radian (in M.K.S)

Note : Always change degree into radian, if it occurs in numerical problems.

Note : 1 radian = $\frac{360^\circ}{2\pi} \Rightarrow$ π radian = 180º

(v) It is a dimensionless quantity i.e. dimension $[M^0L^0T^0]$

Angular Displacement

Ex.1 A particle completes 1.5 revolutions in a circular path of radius 2 cm. The angular displacement of the particle will be -
(in radian)

(A) 6π (B) 3π

(C) 2π (D) π

Sol. **(D)** We have angular displacement

$$= \frac{\text{linear displacement}}{\text{radius of path}}$$

$$\Rightarrow \Delta\theta = \frac{\Delta S}{r}$$

Here, $\Delta S = n(2\pi r)$

$= 1.5\,(2\pi \times 2 \times 10^{-2}) = 6\pi \times 10^{-2}$

$$\therefore \Delta\theta = \frac{6\pi \times 10^{-2}}{2 \times 10^{-2}} = 3\pi \text{ radian}$$

Hence correct answer is (B)

2. ANGULAR VELOCITY

It is defined as the rate of change of angular displacement of a body or particle moving in circular path.

(i) It is a vector quantity.

(ii) Its direction is same as that of angular displacement i.e. perpendicular to plane of rotation.

Note : If the particle is revolving in the clockwise direction then the direction of angular velocity is perpendicular to the plane downwards. Whereas in case of anticlockwise direction the direction will be upwards.

(iii) Its unit is Radian/sec

(iv) Its dimension is $[M^0L^0T^{-1}]$

Types of Angular Velocity :

2.1 Average Angular Velocity :

$$\vec{\omega}_{av} = \frac{\text{Total angular displacement}}{\text{Total time taken}}$$

2.2 Instantaneous Angular velocity :

The intantaneous angular velocity is defined as the angular velocity at some particular instant.

Instantaneous angular velocity

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Note: Instantaneous angular velocity can also be called as simply angular velocity.

Average Angular Velocity

Ex.2 A particle revolving in a circular path completes first one third of circumference in 2 sec, while next one third in 1 sec. The average angular velocity of particle will be :
(in rad/sec)

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{3}$

(C) $\frac{4\pi}{3}$ (D) $\frac{5\pi}{3}$

Sol. (A) We have $\vec{\omega}_{av} = \frac{\text{Total angular displacement}}{\text{Total time}}$

For first one third part of circle,
angular displacement,

$$\theta_1 = \frac{S_1}{r} = \frac{2\pi r/3}{r}$$

For second one third part of circle,

$$\theta_2 = \frac{2\pi r/3}{r} = \frac{2\pi}{3} \text{ rad}$$

Total angular displacement,

$\theta = \theta_1 + \theta_2 = 4\pi/3$ rad

Total time = 2 + 1 = 3 sec

$$\therefore \vec{\omega}_{av} = \frac{4\pi/3}{3} \text{ rad/s}$$

$$= \frac{4\pi}{6} = \frac{2\pi}{3} \text{ rad/s}$$

Hence correct answer is (A)

Ex.3 The ratio of angular speeds of minute hand and hour hand of a watch is -

(A) 1 : 12 (B) 6 : 1

(C) 12 : 1 (D) 1 : 6

Sol. **(C)** Angular speed of hour hand,

$\omega_1 = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{12 \times 60}$ rad/sec

angular speed of minute hand,

$\omega_2 = \frac{2\pi}{60}$ rad/sec $\Rightarrow \frac{\omega_2}{\omega_1} = \frac{12}{1}$

Hence correct answer is (C).

Instantaneous Angular Velocity

Ex.4 The angular displacement of a particle is given by $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$, where ω_0 and α are constant and ω_0 = 1 rad/sec, α = 1.5 rad/sec^2. The angular velocity at time, t = 2 sec will be (in rad/sec) -

(A) 1 (B) 5

(C) 3 (D) 4

Sol. **(D)** We have $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\Rightarrow \frac{d\theta}{dt} = \omega_0 + \alpha t$

This is angular velocity at time t. Now angular velocity at t = 2 sec will be

$\omega = \left(\frac{d\theta}{dt}\right)_{t=2sec} = \omega_0 + 2\alpha$

= 1 + 2 x 1.5 = 4 rad/sec

Hence correct answer is (D)

3. RELATION BETWEEN LINEAR VELOCITY AND ANGULAR VELOCITY

We have $\omega = \frac{d\theta}{dt} = \frac{d\theta}{ds} \cdot \frac{ds}{dt}$

$= \frac{1}{r} \cdot v$

[$\because$ $d\theta = \frac{ds}{dr}$, angle = $\frac{\text{arc}}{\text{radius}}$

and $v = \frac{ds}{dt}$ = linear velocity]

$\Rightarrow v = r\omega$

In vector form, $\vec{v} = \vec{\omega} \times \vec{r}$

Note :

(i) When a particle moves along a curved path, its linear velocity at a point is along the tangent drawn at that point

(ii) When a particle moves along curved path, its velocity has two components. One along the radius, which increases or decreases the radius and another one perpendicular to the radius, which makes the particle to revolve about the point of observation.

(iii) $\omega = \frac{\Delta\theta}{\Delta t} = \frac{v\sin\theta}{r}$

Linear Velocity & Angular Velocity

Ex.5 A particle moves in a circle of radius 20cm with a linear speed of 10m/s. The angular velocity will be -

(A) 50 rad/s (B) 100 rad/s

(C) 25 rad/s (D) 75 rad/s

Sol. The angular velocity is

$\omega = \frac{v}{r}$

Hence v = 10 m/s

r = 20 cm = 0.2 m,

$\therefore \omega = 50$ rad/s

Hence correct answer is (A)

4. ANGULAR ACCELERATION

The rate of change of angular velocity is defined as angular acceleration.

If $\Delta\omega$ be change in angular velocity in time Δt, then angular acceleration

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt}$$

(i) It is a vector quantity

(ii) Its direction is that of change in angular velocity

(iii) Unit : rad/sec^2

(iv) Dimension : $M^0L^0T^{-2}$

Ex.6 The angular velocity of a particle is given by $\omega = 1.5\,t - 3t^2 + 2$, the time when its angular acceleration decreases to be zero will be -

(A) 25 sec (B) 0.25 sec

(C) 12 sec (D) 1.2 sec

Sol. **(B)** Given that $\omega = 1.5t - 3t^2 + 2$

$\alpha = \frac{d\omega}{dt} = 1.5 - 6t$

When $\alpha = 0$

$\Rightarrow 1.5 - 6t = 0$

$\Rightarrow t = \frac{1.5}{6} = 0.25$ sec

Hence correct answer is (B)

5. RELATION BETWEEN ANGULAR ACCELERATION AND LINEAR ACCELERATION

Linear acceleration = Rate of change of linear velocity

$\Rightarrow a = \frac{dv}{dt}$ ....(i)

Angular acceleration = Rate of change of angular velocity

$\Rightarrow \alpha = \frac{d\omega}{dt}$ ....(ii)

From (i) & (ii)

$\frac{a}{\alpha} = \frac{dv}{d\omega} = \frac{d(r\omega)}{d\omega}$

$= r\frac{d\omega}{d\omega}$ [r is constant] = r

$\Rightarrow a = \alpha r$

In vector form $\vec{a} = \vec{\alpha} \times \vec{r}$

Relation Between Angular Acceleration & Linear Acceleration

Ex.7 A particle is moving in a circular path with velocity varying with time as $v = 1.5t^2 + 2t$. If 2 cm the radius of circular path, the angular acceleration at t = 2 sec will be -

(A) 4 rad/sec^2 (B) 40 rad/sec^2

(C) 400 rad/sec^2 (D) 0.4 rad/sec^2

Sol. **(C)** Given $v = 1.5\ t^2 + 2t$

Linear acceleration a

$= \frac{dv}{dt} = 3t + 2$

This is the linear acceleration at time t

Now angular acceleration at time t

$\alpha = \frac{a}{r} \Rightarrow \alpha = \frac{3t + 2}{2 \times 10^{-2}}$

Angular acceleration at t = 2 sec

$(\alpha)_{at\ t = 2sec} = \frac{3 \times 2 + 2}{2 \times 10^{-2}} = \frac{8}{2} \times 10^2$

$= 4 \times 10^2 = 400$ rad/sec^2

Hence correct answer is (C)

6. EQUATION OF LINEAR MOTION AND ROTATIONAL MOTION

(i)	With constant velocity	a = 0, s = ut	$\alpha = 0$, $\theta = \omega t$
(ii)	With constant acceleration	(i) Average velocity $v_{av} = \frac{v+u}{2}$ (ii) Average acceleration $a_{av} = \frac{v-u}{t}$ (iii) $s = v_{av}\, t = \frac{v+u}{2} t$ (iv) $v = u + at$ (v) $s = ut + \frac{1}{2} at^2$ (vi) $s = vt - \frac{1}{2} at^2$ (vii) $v^2 = u^2 + 2as$ (viii) $S_n = u + \frac{1}{2}(2n-1)a$ displacement in n^{th} sec	(i) Average angular velocity $\omega_{av} = \frac{\omega_1 + \omega_2}{2}$ (ii) Average angular acceleration $a_{av} = \frac{\omega_2 - \omega_1}{t}$ (iii) $\theta = \omega_{av}.\ t = \frac{\omega_1 + \omega_2}{2} t$ (iv) $\omega_2 = \omega_1 + \alpha t$ (v) $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$ (vi) $\theta = \omega_2 t - \frac{1}{2}\alpha t^2$ (vii) $\omega_2^2 = \omega_1^2 + 2\alpha\theta$ (viii) $\theta_n = \omega_1 + \frac{1}{2}(2n-1)\alpha$ Angular displacement in n^{th} sec
(iii)	With variable acceleration	(i) $v = \frac{ds}{dt}$ (ii) $\int ds = \int v\, dt$ (iii) $a = \frac{dv}{dt} = v \frac{dv}{ds}$ (iv) $\int dv = \int a\, dt$ (v) $\int v\, dv = \int a\, ds$	(i) $\omega = d\theta/dt$ (ii) $\int d\theta = \int \omega\, dt$ (iii) $\alpha = \frac{d\omega}{dt} = \omega \frac{d\omega}{d\theta}$ (iv) $\int d\omega = \int \alpha\, dt$ (v) $\int \omega\, d\omega = \int \alpha\, d\theta$

Equations of Rotational Motion

Ex.8 A grind stone starts from rest and has a constant-angular acceleration of 4.0 rad/sec^2.The angular displacement and angular velocity, after 4 sec. will respectively be -

(A) 32 rad, 16 rad/sec (B) 16rad, 32 rad/s

(C) 64rad, 32 rad/sec (D) 32 rad, 64rad/sec

Sol. Angular displacement after 4 sec is

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$= \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 4 \times 4^2$

$= 32$ rad

Angular velocity after 4 sec

$\omega = \omega_0 + \alpha t$

$= 0 + 4 \times 4 = 16$ rad/sec

Hence correct answer is (A)

Relation Between Angular Velocity & Angular Acceleration

Ex.9 The shaft of an electric motor starts from rest and on the application of a torque, it gains an angular acceleration given by $\alpha = 3t - t^2$ during the first 2 seconds after it starts after which $\alpha = 0$. The angular velocity after 6 sec will be -

(A) 10/3 rad/sec (B) 3/10 rad/sec

(C) 30/4 rad/sec (D) 4/30 rad/sec

Sol. **(A)** Given $\alpha = 3t - t^2$

$\Rightarrow \frac{d\omega}{dt} = 3t - t^2$

$\Rightarrow d\omega = (3t - t^2)dt$

$\Rightarrow \omega = \frac{3t^2}{2} - \frac{t^3}{3} + c$

at $t = 0$, $\omega = 0$

$\therefore c = 0$, $\omega = \frac{3t^2}{2} - \frac{t^3}{3}$

Angular velocity at

$t = 2$ sec, $(\omega)_{t = 2 \text{ sec}}$

$= \frac{3}{2}(4) - \frac{8}{3} = \frac{10}{3}$ rad/sec

Since there is no angular acceleration after 2 sec

$\therefore$ The angular velocity after 6 sec remains the same.
Hence correct answer is (A)

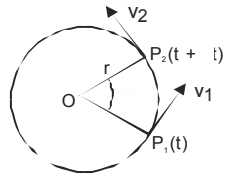
7. CENTRIPETAL ACCELERATION AND CENTRIPETAL FORCE

(i) A body or particle moving in a curved path always moves effectively in a circle at any instant.

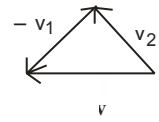
(ii) The velocity of the particle changes moving on the curved path, this change in velocity is brought by a force known as centripetal force and the acceleration so produced in the body is known as centripetal acceleration.

(iii) The direction of centripetal force or acceleration is always towards the centre of circular path.

7.1 Expression for Centripetal Acceleration



(a) Particle moving in circular path of radius r



(b) Vector diagram of velocities

The triangle OP_1P_2 and the velocity triangle are similar

$$\therefore \frac{P_1P_2}{P_1O} = \frac{AB}{AQ}$$

$$\Rightarrow \frac{\Delta s}{r} = \frac{\Delta v}{v} \quad [|\vec{v}_1| = |\vec{v}_2| = v]$$

$$\Rightarrow \Delta v = \frac{v}{r}\Delta s$$

$$\Rightarrow \frac{\Delta v}{\Delta t} = \frac{v}{r}\frac{\Delta s}{\Delta t}$$

$$\Rightarrow \lim_{\Delta t\to 0} \frac{\Delta v}{\Delta t} = \frac{v}{r} \lim_{\Delta t\to 0} \left(\frac{\Delta s}{\Delta t}\right)$$

$$\Rightarrow a_c = \frac{v}{r}\, v = \frac{v^2}{r} = r\omega^2$$

This is the magnitude of centripetal acceleration of particle

(i) It is a vector quantity. In vector form

$$\vec{a}_c = \vec{\omega} \times \vec{v}$$

(ii) The direction of $\vec{a}_c$ would be the same as that of $\Delta\vec{v}$

(iii) Because velocity vector at any point is tangential to the circular path at that point, the acceleration vector acts along radius of the circle at that point and is directed towards the centre. This is the reason that it is called centripetal acceleration.

Centripetal Acceleration

Ex.10 A ball is fixed to the end of a string and is rotated in a horizontal circle of radius 5 m with a speed of 10 m/sec. The acceleration of the ball will be -

(A) 20 m/s^2 (B) 10 m/s^2

(C) 30 m/s^2 (D) 40 m/s^2

Sol. **(A)** We know a = $\frac{v^2}{r}$

Hence v = 10 m/s, r = 5 m

$\therefore$ a = $\frac{(10)^2}{5}$ = 20 m/s^2

Hence correct answer is (A)

Calculation of Centripetal Acceleration by Angular Velocity – Linear Velocity Relation

Ex.11 A body of mass 2 kg lying on a smooth surface is attached to a string 3 m long and then whirled round in a horizontal circle making 60 revolution per minute. The centripetal acceleration will be -

(A) 118.4 m/s^2 (B) 1.18 m/s^2

(C) 2.368 m/s^2 (D) 23.68 m/s^2

Sol. **(A)** Given that the mass of the particle,

m = 2 kg

radius of circle = 3 m

Angular velocity = 60 rev/minute

= $\frac{60 \times 2\pi}{60}$ rad/sec

= 2π rad/sec

Because the angle described during 1 revolution is 2π radian

The linear velocity

v = rω

= 2π × 3 m/s

= 6π m/s

The centripetal acceleration

= $\frac{v^2}{r}$ = $\frac{(6\pi)^2}{3}$ m/s^2

= 118.4 m/s^2

Hence correct answer is (A)

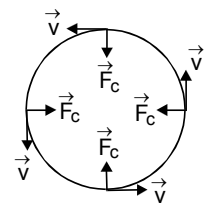
7.2 Expression for Centripetal force

If v = velocity of particle,

r = radius of path

Then necessary centripetal force

F_C = mass × acceleration

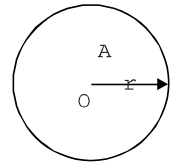


$F_c = m \frac{v^2}{r}$

This is the expression for centripetal force

(i) It is a vector quantity

(ii) In vector form



$\vec{F}_c = -\frac{mv^2}{r}.\hat{r} = -\frac{mv^2}{r^2}\vec{r}$

$= -m\omega^2 r\,\hat{r} = -m\omega^2\,\vec{r} = -m(\vec{v} \times \vec{\omega})$

negative sign indicates direction only

$|\vec{F}_c| = m(\vec{v} \times \vec{\omega})$

(iii) For circular motion :

$|\vec{F}_c| = m(v\,\omega \sin 90^\circ) = mv\omega$

Note :

1. Centripetal force is not a real force. It is only the requirement for circular motion.

2. It is not a new kind of force. Any of the forces found in nature such as gravitational force, electric friction force, tension in string reaction force may act as centripetal force.

Angular Velocity – Centripetal Force Relation

Ex.12 A body of mass 0.1 kg is moving on circular path of diameter 1.0 m at the rate of 10 revolutions per 31.4 seconds. The centripetal force acting on the body is -

(A) 0.2 N (B) 0.4 N

(C) 2 N (D) 4 N

Sol. **(A)** $F = \frac{mv^2}{r} = mr\omega^2$

Here m = 0.10 kg,

r = 0.5 m

and $\omega = \frac{2\pi n}{t} = \frac{2 \times 3.14 \times 10}{31.4}$

= 2 rad/s

$F = 0.10 \times 0.5 \times (2)^2 = 0.2$

Hence correct answer is (A)

Centripetal Force – Angular Velocity Relation

Ex.13 A body of mass 4 kg is moving in a horizontal circle of radius 1 m with an angular velocity of 2 rad/s. The required centripetal force, will be -

(A) 16 N (B) 1.6 N
(C) 16 Dyne (D) 1.6 Dyne

Sol. **(A)** $F = mr\omega^2 = 4 \times 1 \times 2^2 = 16$ N
Hence correct answer is (A)

Centripetal Friction Force Relation

Ex.14 The safe velocity required for scooterist negotiating a curve of radius 200 m on a road with the angle of repose of $\tan^{-1}(0.2)$ will be-

(A) 20 km/hr (B) 200 m/s
(C) 72 km/hr (D) 72 m/s

Sol. **(C)** As the centripetal force is supplied by the frictional force, hence

$$\mu mg = \frac{mv^2}{r} \Rightarrow 0.2 = \frac{v^2}{200 \times 10}$$

$[\theta = \tan^{-1}(0.2) = \tan^{-1}(\mu) \Rightarrow \mu = (0.2)]$

$\Rightarrow v = 20$ m/s

The safe speed is $20 \times \frac{18}{5} = 72$ km/hr

Hence correct answer is (C).

Centripetal Force

Ex.15 A body of mass 4 kg is tied to one end of a rope of length 40 cm and whirled in a horizontal circle. The maximum number of revolutions per minute it can be whirled so that the rope does not snap as the rope can with stand to a tension of 6.4 Newton, will be -

(A) 1.91 (B) 19.1

(C) 191 (D) 1910

Sol. **(B)** Tension in the rope $= mr\omega^2 = mr\ 4\pi^2n^2$

Maximum tension = 6.4 N

$\therefore\ 6.4 = 4 \times 0.4 \times 4 \times \pi^2n^2$

$\therefore$ Number of revolutions per minutes

$= 60/\pi = 19.1$

Hence correct answer is (B)

Ex.16 A certain string which is 1 m long will break, if the load on it is more than 0.5 kg. A mass of 0.05 kg is attached to one end of it and the particle is whirled round a horizontal circle by holding the free end of the string by one hand. The greatest number of revolutions per minute possible without breaking the string will be-

(A) 9.45 (B) 94.5

(C) 99.5 (D) 9.95

Sol. **(B)** Mass of the body m = 0.05 kg ,

Radius of circular path = 1 m

The maximum tension in the string can withstand = 0.5 kg wt = 0.5 × 9.8 N = 4.9 N

Hence the centripetal force required to produce the maximum tension in the string is 4.9 N

i.e. $mr\omega^2 = 4.9 \Rightarrow \omega^2 = \dfrac{4.9}{mr} = \dfrac{4.9}{0.05 \times 1} = 98$

$\Rightarrow \omega = \sqrt{98} \Rightarrow 2\pi n = \Rightarrow n = \sqrt{98}$

= 1.1576 rev/sec = 94.5 rev/min

Hence correct answer is (B)

8. TYPE OF CIRCULAR MOTION

8.1 Uniform circular motion

8.2 Non Uniform Circular Motion :

8.1 Uniform Circular Motion :

If m = mass of body,

r = radius of circular orbit,

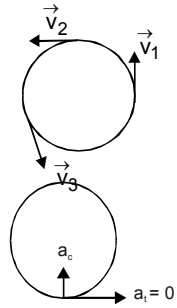
v = magnitude of velocity

a_c = centripetal acceleration,

a_t = tangential acceleration

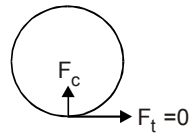
In uniform circular motion :

(i) $|\vec{v}_1| = |\vec{v}_2| = |\vec{v}_3|$ = constant
i.e. speed is constant



(ii) As $|\vec{v}|$ is constant
so tangential acceleration
$a_t = 0$

(iii) Tangential force $F_t = 0$



(iv) Total acceleration

$a = \sqrt{a_c^2 + a_t^2} = a_c = \frac{v^2}{r}$ (towards the centre)

Note:

(i) Because F_c is always perpendicular to velocity or displacement, hence the work done by this force will always be zero.

(ii) Circular motion in horizontal plane is usually uniform circular motion.

(iii) There is an important difference between the projectile motion and circular motion.
In projectile motion, both the magnitude and the direction of acceleration (g) remain constant, while in circular motion the magnitude remains constant but the direction continuously changes.
Hence equations of motion are not applicable for circular motion.
Remember that equations of motion remain valid only when both the magnitude & direction of acceleration are constant.

8.1.1 Hint to solve numerical problem :

(i) Write down the required centripetal force

(ii) Draw the free body diagram of each component of system.

(iii) Resolve the forces acting on the rotating particle along radius and perpendicular to radius

(iv) Calculate net radial force acting towards centre of circular path.

(v) Make it equal to required centripetal force.

(vi) For remaining components see according to question.

Centripetal Force

Ex.17 A body of mass m is attached with a string of length ℓ. If it is whirled in a horizontal circular path with velocity v. The tension in the string will be -

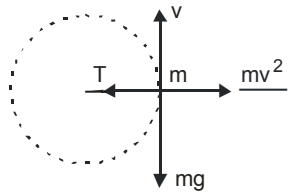
(A) $mv^2\ell$ (B) $\frac{mv^2}{\ell}$

(C) $\frac{m\ell}{v^2}$ (D) $\frac{mv^2}{2\ell}$

Sol. **(B)** Required centripetal force ,

$F_c = \frac{mv^2}{\ell}$

Here centripetal force is provided by the tension in the string



$T = F_c = \frac{mv^2}{\ell}$

Hence correct answer is (B)

Orbital Velocity of Satellite

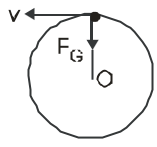
Ex.18 A satellite of mass m is revolving around the earth of mass M in circular orbit of radius r. The orbital velocity of the satellite will be -

(A) $\sqrt{\frac{GM}{r}}$ (B) $\sqrt{\frac{Gm}{r}}$

(C) $\sqrt{\frac{GM}{mr}}$ (D) $\sqrt{\frac{Gm}{Mr}}$

Sol. The required centripetal force,

$F_C =$ (towards the centre)



Net force towards the centre,

$F_G = \frac{GMm}{r^2}$

(This force will provide required centripetal force)

Therefore $F_C = F_G$

$\Rightarrow \frac{mv^2}{r} = \frac{GMm}{r^2}$

$\Rightarrow v = \sqrt{\frac{GM}{r}}$

Hence correct answer is (A)

Note :

(i) From above example we see that orbital velocity of a body is independent to its mass

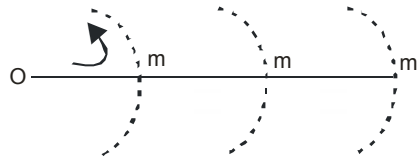
(ii) If we are asked to find out time period of above body then time period can be calculated as

$T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r^3}{GM}}$

$T^2 \propto r^3$ this is **Kepler's law.**

Centripetal Force

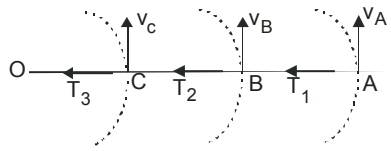
Ex.19 Three identical particles are connected by three strings as shown in fig. These particles are revolving in a horizontal plane. The velocity of outer most particle is v. Then $T_1 : T_2 : T_3$ will be - (Where T_1 is tension in the outer most string etc.)



(A) 3 : 5 : 7 (B) 3 : 5 : 6

(C) 3 : 4 : 5 (D) 7 : 5 : 3

Sol. **(B)** **For A :**



Required centripetal force

$= \frac{mv_A^2}{3}$

(net force towards centre = T_1)

This will provide required centripetal force particle at A,

$\therefore T_1 = \frac{mv_A^2}{3}$

For B :

Required centripetal force

$= \frac{m(v_B^2)}{2\ell}$

Remember ω i.e. angular velocity, of all the particles is same

$\therefore \omega = \frac{v_A}{3\ell} = \frac{v_B}{2\ell} = \frac{v_C}{\ell}$

Note:

When a system of particles rotates about an axis, the angular velocity of all the particles will be same, but their linear velocity will be different, because of different distances from axis of rotation i.e. $v = r\omega$.

Thus for B, centripetal force

$= \frac{2mv_A^2}{9\ell}$

Net force towards the centre

$T_2 - T_1 = \frac{2mv_A^2}{9\ell}$

$\Rightarrow T_2 = \frac{2mv_A^2}{9\ell} + T_1 = \frac{5mv_A^2}{9}$

(Putting value of T_1)

For C :

Centripetal force. $\frac{mv_C^2}{3} = \frac{mv_A^2}{9}$

Net force towards centre $= T_3 - T_2$

$\therefore T_3 - T_2 = \frac{mv_A^2}{9}$

$\Rightarrow T_3 = \frac{mv_A^2}{9} + T_2$

$T_3 = \frac{6mv_A^2}{9}$

(on putting value of T_2)

Now $T_1 : T_2 : T_3 = \frac{1}{3} : \frac{5}{9} : \frac{6}{9} = 3 : 5 : 6$

Note:

It is to be pondered from the above example that as the velocity is increased continuously, the innermost string will break first i.e. $T_3 > T_2 > T_1$

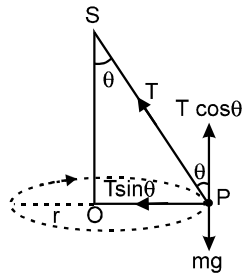
Hence correct answer is (B)

8.1.2 Motion In Horizontal Circle : Conical pendulum

This is the best example of uniform circular motion A conical pendulum consists of a body attached to a string, such that it can revolve in a horizontal circle with uniform speed. The string traces out a cone in the space.

(i) The force acting on the bob are

(a) Tension T (b) weight mg



(ii) The horizontal component T sin θ of the tension T provides the centripetal force and the vertical component T cos θ balances the weight of bob

$\therefore T \sin\theta = \frac{mv^2}{r}$

and $T \cos\theta = mg$

From these equation

$T = mg\sqrt{1+\frac{v^4}{r^2g^2}}$ ....(i)

and $\tan\theta = \frac{v^2}{rg}$ ....(ii)

Also if h = height of conical pendulum

$\tan\theta = \frac{OP}{OS} = \frac{r}{h}$ ....(iii)

From (ii) & (iii),

$\omega^2 = \frac{v^2}{r^2} = \frac{g}{h}$

The time period of revolution

$T = 2\pi \sqrt{\frac{h}{g}} = 2\pi \sqrt{\frac{\ell\cos\theta}{g}}$

[where OS = ℓ]

Motion of Particle in Horizontal Circle

Ex.20 A particle describes a horizontal circle on the smooth surface of an inverted cone. The height of the plane of the circle above the vertex is 9.8 cm. The speed of the particle will be -

(A) 9.8 m/s (B) 0.98 m/s

(C) 0.098 m/s (D) 98 m/s

Sol. **(B)** The force acting on particle are

(i) Weight mg acting vertically downward

(ii) Normal reaction N of the smooth surface of the cone.

(iii) Reaction of the centripetal force $\frac{mv^2}{r}$ acting radially outwards.

Resolving N into horizontal and vertical components we obtain

$N\cos\theta = \frac{mv^2}{r}$ and $N\sin\theta = mg$

$\Rightarrow \frac{N\sin\theta}{N\cos\theta} = \frac{mg}{mv^2/r}$

$\Rightarrow \tan\theta = \frac{rg}{v^2}$

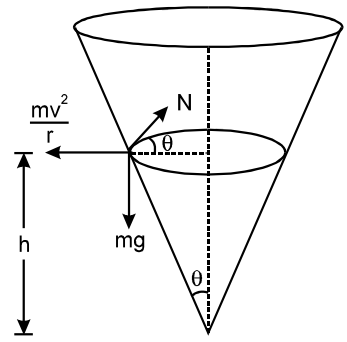
But $\tan\theta = \frac{r}{h}$

$\therefore \frac{r}{h} = \frac{rg}{v^2}$

$\Rightarrow v = \sqrt{hg} = \sqrt{9.8 \times 9.8 \times 10^{-2}}$

$= 0.98$ m/s

Hence correct answer is (B)



Ex.21 A string of length 1 m is fixed at one end and carries a mass of 100 gm at the other end. The string makes $2/\pi$ revolutions per second about a vertical axis through the fixed end. The angle of inclination of the string

with the vertical, and the linear velocity of the mass will respectively be - (in M.K.S. system)

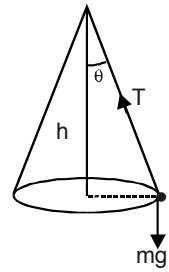
(A) 52°14', 3.16 (B) 50°14', 1.6

(C) 52°14', 1.6 (D) 50°14', 3.16

Sol. **(A)** Let T be the tension, θ the angle made by the string with the vertical through the point of suspension.

The time period

$$t = 2\pi\sqrt{\frac{h}{g}} = \frac{1}{\text{frequency}} = \pi/2$$



Therefore $\omega = \sqrt{\frac{g}{h}} = 4$

$$\Rightarrow \frac{h}{g} = \frac{1}{16}$$

$$\cos\theta = \frac{h}{\ell} = \frac{g}{16} = 0.6125 \Rightarrow \theta = 52°\ 14'$$

Linear velocity

$= (\ell \sin\theta)\omega = 1 \times \sin 52º\ 14' \times 4 = 3.16$ m/s

Hence correct answer is (A)

8.2 Non-uniform Circular Motion :

(i) In non-uniform circular motion :

$|\vec{v}| \neq$ constant $\omega \neq$ constant

i.e. speed $\neq$ constant

i.e. angular velocity $\neq$ constant

(ii) If at any instant

v = magnitude of velocity of particle

r = radius of circular path

ω = angular velocity of particle,

then $v = r\omega$

(iii) Tangential acceleration :

$$a_t = \frac{dv}{dt}$$

where $v = \frac{ds}{dt}$ and s = arc - length

(iv) Tangential force :

$F_t = ma_t$

(v) Centripetal force :

$$F_c = \frac{mv^2}{r} = m\omega^2 r$$

(vi) Net force on the particle :

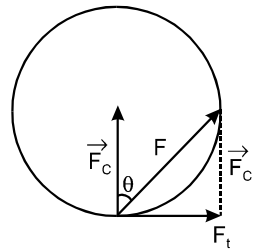
$$\vec{F} = \vec{F}_c + \vec{F}_t$$

$$\Rightarrow F = \sqrt{F_c^2 + F_t^2}$$

If θ is the angle made by [Note angle between F_c and F_t is 90°] F with F_c, then

$$\tan\theta = \frac{F_t}{F_c}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left[\frac{F_t}{F_c}\right]$$



Angle between F & F_t is (90° – θ)

(vii) Net acceleration towards the centre
= centripetal acceleration

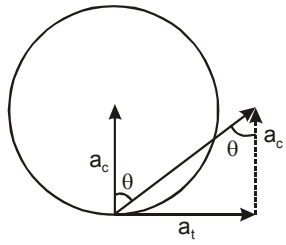
$$\Rightarrow a_c = \frac{v^2}{r} = \omega^2 r = \frac{F_c}{m}$$

(viii) Net acceleration,

$$a = \sqrt{a_c^2 + a_t^2} = \frac{F_{net}}{m}$$

The angle made by 'a' with a_c,

$$\tan\theta = \frac{a_t}{a_c} = \frac{F_t}{F_c}$$



Special Note :

(i) In both uniform & non-uniform circular motion F_c is perpendicular to velocity ; so work done by centripetal force will be zero in both the cases.

(ii) In uniform circular motion $F_t = 0$, as $a_t = 0$, so work done will be zero by tangential force.

But in non-uniform circular motion $F_t \neq 0$, thus there will be work done by tangential force in this case.

Rate of work done by net force in non-uniform circular motion = Rate of work done by tangential force

$$\Rightarrow P = \frac{dW}{dt} = \vec{F}_t \cdot \vec{v} = \vec{F}_t \cdot \frac{d\vec{x}}{dt}$$

Particle's Circular Motion with Variable Velocity

Ex.22 A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$, where k is a constant. The power delivered to the particle by the forces acting on it will be -

(A) mk^2t^2r (B) $mk^2r^2t^2$

(C) $m^2k^2t^2r^2$ (D) mk^2r^2t

Sol. **(D)** Centripetal acceleration,

$$a_c = \frac{v^2}{r} = k^2 rt^2$$

$\therefore$ Variable velocity

$$v = \sqrt{k^2r^2t^2} = k\,r\,t$$

The force causing the velocity to varies

$$F = m\,\frac{dv}{dt} = m\,k\,r$$

The power delivered by the force is,

$P = Fv = mkr \times krt = mk^2r^2t$

Hence correct answer is (D)

Relation between Centripetal & Tangential Acceleration

Ex.23 A car is moving in a circular path of radius 100 m with velocity of 200 m/sec such that in each sec its velocity increases by 100 m/s, the net acceleration of car will be -

(in m/sec)

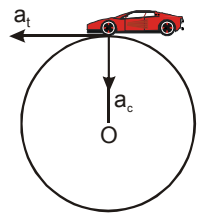
(A) $100\sqrt{17}$ (B) $10\sqrt{7}$

(C) $10\sqrt{3}$ (D) $100\sqrt{3}$

Sol. **(A)** We know centripetal acceleration

$$a_c = \frac{(\text{tangential velocity})^2}{\text{radius}}$$

$= \frac{(200)^2}{100} = 400 \text{ m/sec}^2$



Tangential acceleration

$a_t = 100 \text{ m/sec}^2$ (given)

$\therefore a_{net} = \sqrt{a_c^2 + a_t^2 + 2a_c a_t \cos 90^o}$

$= \sqrt{a_c{}^2 + a_t{}^2}$

$= \sqrt{(400)^2 + (100)^2}$

$= 100\sqrt{17} \text{ m/s}^2$

[Remember the angle between a_t i.e. the tangential acceleration and a_c i.e. the radial acceleration, is always 90º]

Hence correct answer is (A)

Non Uniform Circular Motion

Ex.24 The kinetic energy of a particle moving along a circle of radius R depends on distance covered (s) as $T = as^2$, where a is constant. The force acting on the particle as a function of s will be –

(A) $2as\left[1+\frac{s^2}{R^2}\right]^{1/2}$ (B) $\frac{2as}{R}$

(C) $2as\sqrt{s^2+R^2}$ (D) $\sqrt{\frac{2as}{R}}$

Sol. **(A)** The kinetic energy

$T = as^2$

$\Rightarrow \frac{1}{2}mv^2 = as^2$

$\therefore \frac{mv^2}{R} = \frac{2as^2}{R}$

$\therefore$ Centripetal force or Radial force,

$F_c = \frac{2as^2}{R}$ .... (1)

$\therefore$ Further $mv^2 = 2as^2$

$\Rightarrow v = \sqrt{\frac{2a}{m}}\, s$ .... (2)

$\Rightarrow \frac{dv}{dt} = \sqrt{\frac{2a}{m}}\, \frac{ds}{dt}$

$= \sqrt{\frac{2a}{m}}\, v$ .... (3)

Using (2) and (3) gives tangential acceleration,

$a_t = \frac{dv}{dt} = \sqrt{\frac{2a}{m}}.v$

$= \left(\sqrt{\frac{2a}{m}}\right)^2 s = \frac{2a}{m}s$

$\Rightarrow m\, a_t = 2as$

$\therefore$ Tangential force,

$F_t = ma_t = 2as$

As centripetal and tangential force are mutually perpendicular, therefore

Total Force, $F = \sqrt{F_c^2 + F_t^2}$

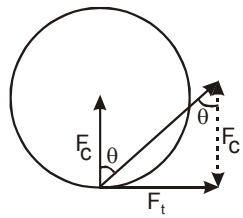
$= \sqrt{\left(\frac{2as^2}{R}\right)^2 + (2as)^2}$

$= 2as\ \sqrt{\frac{s^2}{R^2} + 1}$

Hence correct answer is (A)

Note:

In the above example the angle made by F from the centripetal acceleration will be θ



$\tan\theta = \frac{F_t}{F_c} = \frac{2as}{2as^2/R} = \frac{R}{s}$

Motion in Vertical Circle: Motion of a body suspended by string :

This is the best example of non-uniform circular motion.

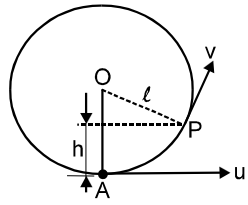
When the body rises from the bottom to the height h apart of its kinetic energy converts into potential energy

Total mechanical energy remains conserved

Total (P.E. + K.E.) at A = Total (P.E. + K.E.) at P

$$\Rightarrow 0 + \frac{1}{2}mu^2 = mgh + \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{u^2 - 2gh} = \sqrt{u^2 - 2g\ell(1-\cos\theta)}$$



[Where ℓ is length of the string]

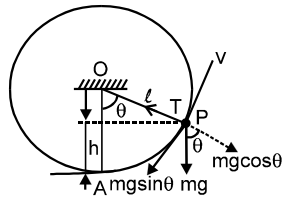
Tension at a point P :

(i) At point P **required centripetal force**

$$= \frac{mv^2}{\ell}$$

(a) Net force towards the centre :

T – mg cos θ, which provides required centripetal force.



$$\therefore T - mg\cos\theta = \frac{mv^2}{\ell}$$

$$T = m\left[g\cos\theta + \frac{v^2}{\ell}\right]$$

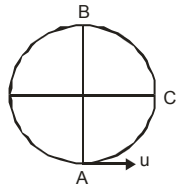
$$= \frac{m}{\ell}\left[u^2 - g\ell\,(2 - 3\cos\theta)\right]$$

(b) Tangential force for the motion

$F_t = mg \sin\theta$

This force retards the motion

(ii) Results :



(a) Tension at the lowest point A :

$$T_A = \frac{mv_A^2}{\ell} + mg$$

(Here $\theta = 0°$)

$$T_A = \frac{mu^2}{\ell} + mg$$

(b) Tension at point B :

$$T_B = \frac{mv_B^2}{\ell} - mg$$

$$T_B = \frac{mu^2}{\ell} - 5mg$$

($\because \theta = 180°$)

(c) Tension at point C :

$$T_C = \frac{mv_C^2}{\ell}$$

$$T_C = \frac{mu^2}{\ell} - 2mg$$

(Here $\theta = 90°$)

Thus we conclude that

$T_A > T_C > T_B$

and also

$T_A - T_B = 6\ mg$

$T_A - T_C = 3\ mg$

$T_C - T_B = 3\ mg$

(iii) Cases :

(a) If $u > \sqrt{5g\ell}$

In this case tension in the string will not be zero at any of the point, which implies that the particle will continue the circular motion.

(b) If $u = \sqrt{5g\ell}$

In this case the tension at the top most point (B) will be zero, which implies that the particle will just complete the circular motion.

(c) **Critical Velocity :** The minimum velocity at which the circular motion is possible

The critical velocity at A = $\sqrt{5g\ell}$

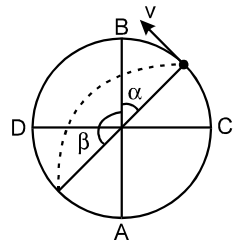
The critical velocity at B = $\sqrt{g\ell}$

The critical velocity at C = $\sqrt{3g\ell}$

Also $T_A = 6$ mg, $T_B = 0$, $T_C = 3$ mg

(d) If $\sqrt{2g\ell} < u < \sqrt{5g\ell}$

In this case particle will not follow circular motion. Tension in string becomes zero somewhere between points C & B whereas velocity remain positive. Particle leaves circular path and follow parabolic trajectory



(e) If $u = \sqrt{2g\ell}$

In this case both velocity and tension in the string becomes zero between A and C and particle will oscillate along semi-circular path.

(f) If $u < \sqrt{2g\ell}$

The velocity of particle remains zero between A and C but tension will not be zero and the particle will oscillate about the point A.

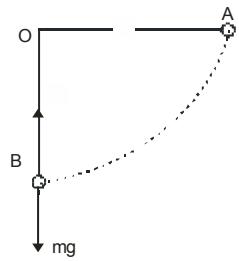
Velocity at Minimum Point in Vertical Circular Motion

Ex.25 A particle of mass m tied with a string of length ℓ is released from horizontal as shown in fig. The velocity at the lowest portion will be -

(A) $\sqrt{g\ell}$ (B) $\sqrt{2g\ell}$

(C) $\frac{1}{2}\sqrt{g\ell}$ (D) $\frac{1}{\sqrt{2}}g\ell$

Sol. **(B)** Suppose v be the velocity of particle at the lowest position B.
According to conservation of energy
(K.E. + P.E.) at A = (K.E. + P.E.) at B



$\Rightarrow 0 + mg\ell = \frac{1}{2}mv^2 + 0$

$\Rightarrow v = \sqrt{2g\ell}$

Hence correct answer is (B)

Maximum Velocity in Vertical Circular Motion

Ex.26 A 4 kg balls is swing in a vertical circle at the end of a cord 1 m long. The maximum speed at which it can swing if the cord can sustain maximum tension of 163.6 N will be -

(A) 6 m/s (B) 36 m/s
(C) 8 m/s (D) 64 m/s

Sol. **(A)** Maximum tension $T = \frac{mv^2}{r} + mg$

$\therefore \frac{mv^2}{r} = T - mg$

or $\frac{4v^2}{1} = 163.6 - 4 \times 9.8$

$\Rightarrow v = 6$ m/s

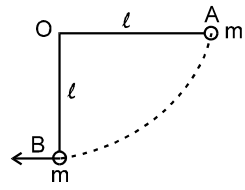
Hence correct answer is (A)

Tension at Minimum Point in Vertical Circular Motion

Ex.27 The string of a pendulum is horizontal. The mass of the bob is m. Now the string is released. The tension in the string in the lowest position is -

(1) 1 mg (2) 2 mg
(3) 3 mg (4) 4 mg

Sol. **(C)** The situation is shown in fig. Let v be the velocity of the bob at the lowest position. In this position the P.E. of bob is converted into K.E. hence -



$mg\ell = \frac{1}{2}mv^2$

$\Rightarrow v^2 = 2g\ell$ ....(1)

If T be the tension in the string,

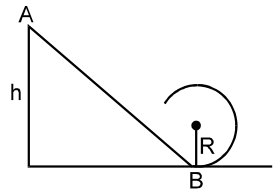
then $T - mg = \frac{mv^2}{\ell}$ ....(2)

From (1) & (2) T = 3 mg

Hence correct answer is (C)

Critical Velocity at Minimum Point in Vertical Circular Motion

Ex.28 A ball is released from height h as shown in fig. Which of the following condition hold good for the particle to complete the circular path?



(A) $h \le \frac{5R}{2}$ (B) $h \ge \frac{5R}{2}$

(C) $h < \frac{5R}{2}$ (D) $h > \frac{5R}{2}$

Sol. **(B)** According to law of conservation of energy

(K.E + P.E.) at A = (K.E + P.E) at B

$\Rightarrow 0 + mgh = \frac{1}{2}mv^2 + 0$

$\Rightarrow v = \sqrt{2gh}$

But velocity at the lowest point of circle,

$v \ge \sqrt{5gR} \Rightarrow \sqrt{2gh} \ge \sqrt{5gR} \Rightarrow h \ge \frac{5R}{2}$

Hence correct answer is (B)

Critical Velocity at Maximum Point in Vertical Circular Motion

Ex.29 The roadway bridge over a canal is the form of an arc of a circle of radius 20 m. What is the minimum speed with which a car can cross the bridge without leaving contact with the ground at the highest point (g = 9.8 m/s^2)

(A) 7 m/s (B) 14 m/s

(C) 289 m/s (D) 5 m/s

Sol. **(B)** The minimum speed at highest point of a vertical circle is given by

$v_c = \sqrt{rg} = \sqrt{20 \times 9.8} = 14$ m/s

Hence correct answer is (B)

Maximum Periodic time in Vertical Circular Motion

Ex.30 A cane filled with water is revolved in a vertical circle of radius 0.5 m and the water does not fall down. The maximum period of revolution must be –

(A) 1.45 (B) 2.45

(C) 14.15 (D) 4.25

Sol. **(A)** The speed at highest point must be

$v > \sqrt{gr}$, $v = r\omega = r\,\frac{2\pi}{T}$

$\therefore\ r\,\frac{2\pi}{T} > \sqrt{rg}$

$T < \frac{2\pi r}{\sqrt{rg}} < 2\pi\sqrt{\frac{r}{g}} < 2\pi\sqrt{\frac{0.5}{9.8}} < 1.4$ sec

Maximum period of revolution = 1.4 sec

Hence correct answer is (A)

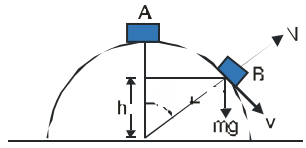
Vertical Semicircular Motion

Ex.31 A particle of mass m slides down from the vertex of semi-hemisphere, without any initial velocity. At what height from horizontal will the particle leave the sphere-

(A) $\frac{2}{3}R$ (B) $\frac{3}{2}R$

(C) $\frac{5}{8}R$ (D) $\frac{8}{5}R$

Sol. **(A)** Let the particles leaves the sphere at height h,



$\frac{mv^2}{R} = mg\cos\theta - N$

When the particle leaves the sphere

i.e. N = 0

$\frac{mv^2}{R} = mg\cos\theta$

$\Rightarrow v^2 = gR\cos\theta$ ....(1)

According to law of conservation of energy

(K.E. + P.E.) at A = (K.E. + P.E.) at B

$\Rightarrow 0 + mgR = \frac{1}{2}mv^2 + mgh$

$\Rightarrow v^2 = 2g(R - h)$ ....(2)

From (1) & (2) $h = \frac{2}{3}R$

Also $\cos\theta = \frac{2}{3}$

Hence correct answer is (A)

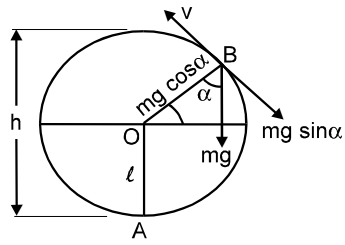
Vertical Circular Motion

Ex.32 A body of mass m tied at the end of a string of length ℓ is projected with velocity $\sqrt{4}$ at what height will it leave the circular path -

(A) $\frac{5}{3}\ell$ (B) $\frac{3}{5}\ell$

(C) $\frac{1}{3}\ell$ (D) $\frac{2}{3}\ell$

Sol. **(A)** Let the body will have the circular path at height h above the bottom of circle from figure



$\frac{mv^2}{\ell} = T + mg\cos\alpha$

On leaving the circular path

$T = 0$

$\therefore \frac{mv^2}{\ell} = mg\cos\alpha$

$\Rightarrow v^2 = g\ell\cos\alpha$ ....(1)

According to law of conservation of energy

(K.E. + P.E.) at A = (K.E. + P.E.) at B

$\Rightarrow 0 + 2mg\ell = \frac{1}{2}mv^2 + mgh$

$\Rightarrow v^2 = 2g(2\ell - h)$ ....(2)

From (1) & (2) $h = \frac{5}{3}\ell$

Also $\cos\alpha = \frac{h-\ell}{\ell}$

Hence correct answer is (A)

9. BANKING OF TRACKS

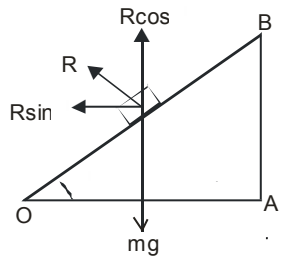
When a vehicle moves round a curve on the road with sufficient speed, there is a tendency of over turning for the vehicle. To avoid this the road is given a slope rising outwards. The phenomenon is known as banking

(i) Let there be vehicle on a road having slope θ. R = normal reaction of the ground

Horizontal component	Vertical component
$R\sin\theta$ It provides necessary centripetal force $R\sin\theta = \frac{mv^2}{r}$	$R\cos\theta$ It balances the weight of the vehicle $R\cos\theta = mg$

$\Rightarrow \tan\theta = \frac{v^2}{rg}$

This equation gives the angle of banking required.



Conditions for skidding and overturning :

Let there be a car moving on a road moving on a curved path.

$2a \rightarrow$ distance between the wheels

$h \rightarrow$ height of centre of gravitiy above the ground

The force acting on car are.

(i) Weight of car W = mg acting downward

(ii) Normal reactions of ground R_a and R_b on inner and outer wheels respectively

(iii) The force of friction μR_a and μR_b

Condition for skidding :

If r is radius of circular path, for equilibrium

$$W = mg = R_a + R_b \;\&\; \mu R_a + \mu R_b = \frac{mv^2}{r}$$

$$\Rightarrow \mu (R_a + R_b) = \frac{mv^2}{r}$$

$$\Rightarrow \mu \; mg = \frac{mv^2}{r}$$

This gives maximum speed for skidding,

$$v_{max} = \sqrt{\mu rg}$$

Condition for overturning :

Taking moments about B, we get,

$$R_a \cdot 2a + \frac{mv^2}{r} h - mg\, a = 0$$

$$\Rightarrow R_a = \frac{mg}{2}\left[1 - \frac{v^2 h}{rag}\right]$$

If we take moments about A, we get

$$\Rightarrow R_b = \frac{mg}{2}\left[1 + \frac{v^2 h}{rag}\right]$$

We know that R_b is always positive while R_a decreases as speed of the car increases.

When $\frac{v^2h}{rag} = 1$

$\Rightarrow R_a = 0$

i.e. inner wheel tends to loose contact with the earth.

When $\frac{v^2h}{rag} > 1$

$\Rightarrow R_a$ = Negative

i.e. the car overturns outwards.

Thus the maximum speed for no overturing is given by

$1 - \frac{v^2h}{rag} = 0$

$\Rightarrow v_{max} = \sqrt{\frac{rag}{h}}$

Required Centripetal Force for Motion on Circular Path

Ex.33 A vehicle of mass 1000 kg is moving along a curved both of length 314 m with a speed of 72 km/hr. If it takes a turn of 90°, the centripetal force needed by the vehicle is -

(A) 20 N (B) 200 N

(C) 2000 N (D) 2 N

Sol. As the vehicle has a turn of 90°, the length of $\frac{1}{4}$ the path is the part of the circle of radius r.

Hence length of the path

$= 314 = \frac{2\pi r}{4}$

or $r = \frac{4 \times 314}{2\pi} = 200$ m

Centripetal force, $F_c = \frac{mv^2}{r}$

$= \frac{1000}{200} \times \left(72 \times \frac{5}{18}\right)^2$

= 2000 N

Hence correct answer is (C)

Necessary Condition for Motion on Circular Path

Ex.34 For a heavy vehicle moving on a circular curve of a highway the road bed is banked at an angle θ corresponding to a particular speed. The correct angle of banking of the road for vehicles moving at 60 km/hr will be - (If radius of curve = 0.1 km)

(A) $\tan^{-1}(0.283)$ (B) $\tan^{-1}(2.83)$

(C) $\tan^{-1}(0.05)$ (D) $\tan^{-1}(0.5)$

Sol. **(A)** v = 60 km/hr = $\frac{50}{3}$ m/s

r = 0.1 km = 100m

$\therefore \tan\theta = \frac{v^2}{rg} = 0.283$

$\therefore \theta = \tan^{-1}(0.283)$

Hence correct answer is (A)

Ex.35 A train has to negotiate a curve of radius 400 m. By how much should the outer rail be raised with respect to inner rail for a speed of 48 km/hr. The distance between the rail is 1 m.

(A) 12 m (B) 12 cm

(C) 4.5 cm (D) 4.5 m

Sol. **(C)** We know that $\tan\theta = \frac{v^2}{rg}$ ..... (1)

Let h be the relative raising of outer rail with respect to inner rail. Then

$\tan\theta = \frac{h}{\ell}$ ..... (2)

(ℓ = separation between rails)

From (1) & (2) , $h = \frac{v^2}{rg} \times \ell$

Hence v = 48 km/hr = $\frac{120}{9}$ m/s,

(r = 400 m, ℓ = 1m),

$\therefore h = \frac{(120/9)^2 \times 1}{400 \times 9.8} = 0.045$ m = 4.5 cm

Hence correct answer is (C)

POINTS TO REMEMBER

1. Centripetal force does not increase the kinetic energy of the particle moving in circular path, hence the work done by the force is zero.
2. Centrifuges are the apparatuses used to separate small and big particles from a liquid.
3. The physical quantities which remain constant for a particle moving in circular path are speed, kinetic energy and angular momentum.
4. If a body is moving on a curved road with speed greater than the speed limit, the reaction at the inner wheel disappears and it will leave the ground first.
5. On unbanked curved roads the minimum radius of curvature of the curve for safe driving is $r = v^2/\mu g$, where v is the speed of the vehicle and μ is small.
6. If r is the radius of curvature of the speed breaker, then the maximum speed with which the vehicle can run on it without leaving contact with the ground is $v = \sqrt{(gr)}$
7. While taking a turn on the level road sometimes vehicles overturn due to centrifugal force.
8. If h is the height of centre of gravity above the road, a is half the wheel base then for road safety $\frac{mv^2}{r}.h < mg \cdot a$, $\therefore$ Minimum safe speed for no overturning is $v = \sqrt{(gar/h)}$.
9. On a rotating platform, to avoid the skidding of an object placed at a distance r from axis of rotation, the maximum angular velocity of the platform, $\omega = \sqrt{(\mu g/r)}$, where μ is the coefficient of friction between the object and the platform.
10. If an inclined plane ends into a circular loop of radius r, then the height from which a body should slide from the inclined plane in order to complete the motion in circular track is $h = 5r/2$.
11. Minimum velocity that should be imparted to a pendulum to complete the vertical circle is $\sqrt{(5g\ell)}$, where ℓ is the length of the pendulum.
12. While describing a vertical circle when the stone is in its lowest position, the tension in the string is six times the weight of the stone.

13. The total energy of the stone while revolving in vertical circle is (5/2) mgℓ.

14. When the stone is in horizontal position then the tension in the string is 3mg and the velocity of the stone is $\sqrt{(3g\ell)}$.

15. If the velocity of the stone at the highest point is X mg, then the tension at the lowest point will be (X + 6)mg.

16. If a body of mass m is tied to a string of length ℓ and is projected with a horizontal velocity u such that it does not complete the motion in the vertical circle, then

(a) the height at which the velocity vanishes is

$$h = \frac{u^2}{2g}$$

(b) the height at which the tension vanishes is

$$h = \frac{u^2 + g\ell}{3g}$$

17. K.E. of a body moving in horizontal circle is same throughout the path but the K.E. of the body moving in vertical circle is different at different places.