Fluid Mechanics

HOOKE'S LAW

If deformation is small, the stress in a body is proportional to the corresponding strain.

In the elasticity limit stress and strain

Stress/strain = Const. = Modulus of elasticity.

Young's Modulus:

If a rod is stretched by equal and opposite forces F each, a tensile stress $\frac{F}{A}$ is produced in the rod where A is the area of cross-section. The length of the rod increases from its natural value L to L + ΔL, tensile strain is $\frac{\Delta L}{L}$.

Strain = $\frac{L + \Delta L - L}{L}$

By Hooke's law, for small deformations,

$$Y = \frac{\text{Tensile stress}}{\text{Tensile strain}}$$

is a constant for the given material. This ratio of tensile stress over tensile strain is called Young's modulus for the material. In the situation described above, the Young's modulus is

$$Y = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

$$= \frac{FL}{A\Delta L}$$

$$\Delta L = \frac{FL}{AY}$$

If $Y \uparrow \Rightarrow \Delta L \downarrow$

Y is a characteristic of solid.

Ex. **A load of 4.0 kg is suspended from a ceiling through a steel wire of length 20 m and radius 2.0 mm. It is found that the length of the wire increases by 0.031 mm as equilibrium is achieved. Find Young's modulus of steel. Take g = 3.1 π m/s^2.**

Solution:

The longitudinal stress

$$= \frac{(4.0)(3.1\pi)}{\pi\left(2.0 \times 10^{-3}\right)^2}$$

$= 3.1 \times 10^6$ N/m^2

The longitudinal strain

$$= \frac{0.31 \times 10^{-3}}{2.0}$$

$= 0.0155 \times 10^{-3}$

$$Y = \frac{\text{long stress}}{\text{long strain}}$$

Thus,

$$Y = \frac{3.1 \times 10^{6}}{0.0155 \times 10^{-3}}$$

$= 2.0 \times 10^{11}\ N/m^2$.

PRESSURE IN A FLUID:

FLUID

Matter is broadly divided into three categories, solid, liquid and gas. The intermolecular forces are strong in solids, so that the shape and size of solids do not easily change. This force is comparatively less in liquids and so the shape is easily changed:

Although the shape of a liquid can be easily changed, the volume of a given mass of a liquid is not so easy to change. It needs quite a good effort to change the density of liquids. In gases, the intermolecular forces are very small and it is simple to change both the shape and the density of a gas. Liquids and gases together are called fluids, i.e., that which can flow.

In this chapter we shall area liquids. We shall assume that the liquids are incompressible and non-viscous.

The first condition means that the density of the liquid is independent of the variations in pressure and always remains constant. The second condition means that parts of the liquid in contact do not exert any tangential force on each other. The force by one part of the liquid on the other part is perpendicular to the surface of contact. Thus, there is no friction between the adjacent layers of a liquid.

Pressure in a Fluid:

Consider a point A in the fluid. Imagine a small area ΔS containing the point A. The fluid on one side of the area presses the fluid on the other side and vice-versa. Let the common magnitude of the forces be F. We define the pressure of the fluid at the point A as

$$P = \lim_{\Delta S \to 0} . \frac{F}{\Delta S}$$

For a homogeneous and non viscous fluid, this quantity does not depend on the orientation of ΔS and hence we talk of pressure at a point. For such a fluid, pressure is a scalar quantity having only magnitude.

Unit of Pressure

The SI unit of pressure is N/m^2 called pascal and abbreviated as Pa.

Pascal's Law:

Since pressure difference between two points in a liquid at rest depends only on the difference in vertical height between the points. The difference is in fact ρgZ, where ρ is the density of the density of the liquid (assumed constant) and Z is the difference in vertical height.

If the pressure in a liquid is changed at a particular point, the change is transmitted to the entire liquid without being diminished in magnitude.

As an example,

Suppose a flask fitted with a piston is filled with a liquid as shown in figure. Let an external force F be applied on the piston. If the cross-sectional area of the piston is A, the pressure just below the piston is increased by $\frac{F}{A}$. By Pascal's law, the pressure at any point B will also increase by the same amount $\frac{F}{A}$.

This is because the pressure at B has to be ρgZ more than the pressure at the piston, where Z is the vertical distance of B below the piston. By applying the force we do not appreciably change Z (as the liquid is supposed to be incompressible) and hence the pressure difference remains unchanged. As the pressure difference increased by $\frac{F}{A}$, the pressure at B also increases by the same amount.

Ex. **The area of cross-section of the wider tube shown in figure is 900 cm^2. If the boy standing on the piston weights 45 kg, find the difference in the levels of water in the two tubes.**

Solution:

$$P_A = P_B + h\rho g$$

$$P_A - P_B = h\rho g$$

$$\frac{45\ g}{900 \times 10^{-4}} = h\rho g$$

$$h = \frac{45}{900 \times 10^{-4} \times 1000}$$

$$= \frac{45}{90}$$

$$= \frac{1}{2} \text{ m}$$

= 0.5 m

= 50 cm

Ex. **The area of cross-section of the two arms of a hydraulic press are 1 cm^2 and 10 cm^2 respectively.**

A force of 5N is applied on the water in the thinner arm. What force should be applied on the water in the thicker arm so that the water may remain in equilibrium?

Solution:

In equilibrium, the pressure at the two surfaces A and B should be equal as they lie in the same horizontal level.

If P is atmospheric pressure

$P_A = P_B$

$$P_0 + \frac{5}{1 \text{ cm}^2}$$

$$= P_0 + \frac{F}{10 \text{ cm}^2}$$

F = 50 N

Ex. **An open U-tube of uniform cross-section contains mercury. When 27.2 cm of water is poured into one limb of the tube,**

(a) **how high does the mercury rise in the other limb from its initial level?**

(b) **What is the difference in levels of liquids of the two sides?**

(ρ_w = 1 and ρ_{rig} = 13.6 units)

The situation is:

(a) If water depresses the mercury by y, mercury in the other limb will rise by y above its initial level (as fluid is incompressible)

$$\therefore h_2 = 2y$$

Now if h_1 is the height of water column above A', then as in a liquid, pressure is same at all points in the same level.

$$P_{A'} = P_{C'}$$

or $_s$

$= h_2\rho_2 g + P_0$

or $h_1\rho_1 = h_2\rho_2$

or 27.2 × 1 = 2y × 13.6

y = 1 cm

i.e. mercury rises by 1 cm from its initial level.

(b) The difference of level on two sides

$z = h_1 - h_2$

$= 27.2 - 2 \times 1$

$= 25.2$ cm

i.e. the water level will stand 25.2 cm higher

then the mercury level in the other limb.

Some More Points About Pressure:

For a point at depth h below the surface of a liquid density

ρ, hydrostatic pressure

P is given by

$$P - P_0 + h\rho g$$

where P_0 is the atmospheric pressure. The pressure difference between hydrostatic pressure and atmosphere pressure is called gauge pressure and will be

$$P - P_0 = h\rho g$$

Regarding pressure it is worth noting that:

(1) Even though pressure is produced by a force that has directional properties and is a vector, the pressure itself is a scalar as its direction (always normal to the area considered) is unique and not to be specified.

(2) 1 atm. = 1.01×10^5 Pa

= 1.01 bar

= 760 torr

(3) At a point it acts in all directions

(4) It always acts normal to the fluid boundaries as the ability to flow make fluid unable to sustain a tangential force.

(5) It depends on the depth of the point below the surface (h), nature of liquid (ρ) and accn. due to gravity (g) while it is independent of the amount of liquid, shape of container or cross-sectional area considered.

So if a given liquid is filled in vessels of different shapes to same

height, the pressure at the base in each vessel's will be the same, though the

volume or weight of the liquid in different vessel will be different.

$P_A = P_B = P_C$

but $W_A < W_B < W_C$

(6) In a liquid at same level, the pressure will be the same at all points, if not, due to pressure difference the liquid can not be at rest. This is why the height of liquid is the same in vessels of different shapes containing different amounts of the same liquid at rest when they are communication with each other.

$P_A = P_B = P_C = P_D = P_E$

Also,

$h_A = h_B = h_C = h_D = h_E$

(7) Barometer is used to measure atmospheric pressure while manometer measures pressure difference, i.e., gauge-pressure.

BUOYENCY:

Archimedes discovered that when a body is immersed partly or wholly in a fluid, it is buoyed up with a force equal to the weight of the fluid displaced by the body. This principle is called 'Archimedes' principle and is a necessary consequence of the laws of fluid statics.

When a body is partly or wholly dipped into a fluid, the fluid exerts force on the body due to hydrostatic pressure. At only small portion o the surface of the body, the force exerted by the fluid is perpendicular to the surface and is equal to the pressure at that point multiplied by the area the resultant of all these constant forces is called upthrust or buoyancy.

To determine the magnitude and direction of this force consider a body immersed in a fluid of density σ as shown in fig. The forces on the vertical sides of the body will cancel each other. The top surface of the body will experience a downward force

$F_1 = Ap_1$

$= A\,(h_1\,\sigma g + p_0)$

While the lower face of the body will experience an upward force

$F_2 = Ap_2$

$= A(h_2\sigma g + p_0)$

As $h_2 > h_1$, F_2 will be greater than F_1, so the body will experience a net upward force.

$F = F_2 - F_1$

$= A\sigma g\ (h_2 - h_1)$

If L is the vertical height of the body

$F = A\sigma gL$

$= V\sigma g$

i.e. F = weight of fluid displace by the body.

This force is called upthust or buoyancy and acts vertically upwards (opposite to the weight of the body) through the centre of gravity of displaced fluid (called centre of buoyancy).

Regarding upthrust or buoyancy:

(1) Th $\neq f^n$ [mass, size, density of body]

$= f^n$ [volume of body]

Th $\propto V_{in}$ [volume of body in the fluid]

This is why two bodies of different masses, shapes and sizes may experience same thrust when their volumes inside a fluid are equal.

(2) Th $\propto \sigma$ [Nature of liquid]

(3) Th $\propto g$ [Accn. due to gravity]

(4) Due to upthrust weight of body decreases

i.e., Decrease in weight of the body

= upthrust $V_{in} \sigma g$

= Weight of fluid displaced by the body.

So, if W_0 is the true weight of a body, the weight in a fluid (called apparent weight) will be

$W_{app} = W_0 - Th$

with $Th = V_{in} \sigma g$

Weight of a body and liquid under different situations.

(C) When a (Sinking) solid is suspended from an independent support in a liquid, the weight of liquid will increase by an amount equal to the decrease in weight of solid (i.e., thrust) as for every action there is equal and opposite reaction.

(D) If the solid (sinking or floating) of weight a is placed or suspended in a liquid of weight b as shown in fig. (D), the reading or balance will always be (a – Th) + (b + Th) = a + b

Using Archimedes principle or concept of thrust, we can determine.

(a) The RD of a body

$$RD = \frac{\text{density of body}}{\text{density of water}}$$

$$= \frac{\text{weight of body}}{\text{weight of equal wt of water}}$$

$$\text{i.e., } RD = \frac{\text{wt of body}}{\text{water thrust}}$$

$$= \frac{\text{wt of body}}{\text{loss in wt in water}}$$

$$\text{i.e., } RD = \frac{\text{wt of body in air}}{\text{wt in air} - \text{wt in water}}$$

NOTE:

If the loss in weight of a body in water is a while in a liquid is b

$Th_W = V\, \sigma_W\, g$

$= a$

and

$Th_L = V \sigma_L g$

$= b$

$$\frac{\sigma_L}{\sigma_W} = \frac{Th_L}{Th_W}$$

$$= \frac{\text{loss in wt in liquid}}{\text{loss in wt in water}}$$

$$= \frac{b}{a}$$

$$= \frac{W_{air} - W_{liquid}}{W_{air} - W_{water}}$$

So $\rho = \rho_0 \left(1 - \frac{\Delta p}{B}\right)^{-1}$

$$\simeq \rho_0 \left(\frac{1 + \Delta p}{B}\right)$$

[B] Pressure

If we fill a vessel having a hole with a fluid, fluid flows out of the hole. Now if we cover this hole with a plate, which exactly fits the hole, the plate can remain at rest only if we apply some external force on the plate. This shows that fluid exerts force on the plate to push it outwards.

If Δs is the area of the plate and ΔF is the normal force exerted by the fluid, the pressure at the hole

$$p = \lim_{\Delta S \to 0} \frac{\Delta F}{\Delta s}$$

$$= \frac{dF}{ds}$$

For a point at depth h below the surface of a liquid density ρ, hydrostatic pressure p is given by

$$P - P_0 + h\rho g$$

where P_0 is the atmospheric pressure. The pressure difference between

hydrostatic pressure and atmosphere pressure is called **gauge-pressure** and will be

$$P - P_0 = h\rho g$$

Regarding pressure it is worth noting that:

(1) Even though pressure is produced by a force that has directional properties and is a vector, the pressure itself is a scalar as its direction (always normal to the area considered) is unique and not to be specified.

(2) It has dimensions $[ML^{-1} T^{-2}]$ and SI unit N/m^2 which is specifically called pascal (Pa). Other practical units of pressure are atmosphere, bar and torr (mm of Hg) with

$1 \text{ atm.} = 1.01 \times 10^5 \text{ Pa}$

= 1.01 bar

= 760 torr

(3) At a point it acts in all directions.

If a pressure measuring device is at a given point in a fluid, whatever be its orientation the pressure remains the same [Fig. 10.2]

(4) It always acts normal to the fluid boundaries as the ability to flow make fluid unable to sustain a tangential force.

(5) It depends on the depth of the point below the surface (h), nature of liquid (ρ) and acceleration due to gravity (g) while it is independent of the amount of liquid, shape of container or cross-sectional area considered.

So if a given liquid is filled in vessels of different shapes to same height, the pressure at the base in each vessel's will be the same, though the volume or weight of the liquid in different vessel will be different.

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