CURRENT ELECTRICITY - ELECTRIC CURRENT

The current through any cross-section area is defined as the net charge flowing through the area per unit time.

If a net charge ΔQ crosses an area in the time interval from t to t + Δt, the average electric current through the area during the time Δt is

$$i_{av} = i = \frac{\Delta Q}{\Delta t}$$ ………..(average current)

The current at time t is

$$i = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$ ………(definition of instantaneous current)

The charge that passes through the area in a time interval from 0 to t is

$$Q = \int dQ = \int_0^t i\,dt$$

If i is independent to t, then Q = it

So, the average current during the time interval 0 to t is

$$i_{av} = \frac{Q}{t} = \frac{\int_0^t i\,dt}{t}$$

……[average current]

SI unit of electric current is ampere (A)

1 ampere = 1A = $\frac{1\ \text{coulomb}}{1\ second}$ = 1C/s

= flow of 6.25×10^{18} electrons/s

(i) Electric current has direction; still it is scalar because the current does not obey the law of vector addition. Current is always along the length of the current carrying wire, regardless of whether the wire is straight or curved.

(ii) The direction of current is taken as the direction of motion of positive charged particles, though the negatively charged particles, electrons move in a conducting wire in the direction opposite to the direction of our conventional current.

Electromotive Force (EMF) and Voltage:

Figure is a schematic diagram of an ideal source of emf that maintains a potential difference between the terminals A and B of the device. The terminal A, marked '+' is maintained at higher potential than terminal B, marked '-'. This potential difference is maintained by some internal mechanism that drives the positive charges of the source towards A and the negative charges of the source towards B.

When a positive charge q is moved from $\vec{B}$ to $\vec{A}$ inside the source, the non-electrostatic force F_n does a positive work W_n against the electrostatic for F_e. So, the potential energy associated with the charge increases by an amount equal to $W_n = qV_{AB}$ where $V_{AB} = V_A - V_B$ the potential of A with respect to B.

The work done by the non-electrostatic force F_n per unit charge is called the emf of the source. It is usually represented by ε. Hence,

$$\varepsilon = \frac{W_n}{q} = \frac{qV_{AB}}{Q} = V_{AB}$$

Thus, the emf of a source equals the potential difference between the terminals when the terminals are not connected externally.

S.I. Unit of emf is volt, the unit of potential

1 volt = 1 joule/coulomb

Remember:

1. Actually, it is the negative charges (electrons) that flow in a metallic conductor of an external circuit while in a battery (source) both positive and negative ions move.

2. In the external part of the circuit, free electrons (i.e. negative charge) move from lower potential to higher potential but in internal part of the source negative charges move from higher potential to lower potential and positive charges move from lower potential to higher potential.

Let's see an illustration:

Example 3: In the Bohr's model of hydrogen atom, the electron is assumed to rotate in a circular orbit of radius 5×10^{-11} m, at a speed of 2.2×10^{6} m/s. what is the current associated with electron motion?

Solution:

$$I = \frac{q}{t} = \frac{q}{2\pi R}\left(\because t = \frac{distance}{speed} = \frac{2\pi R}{v}\right)$$

$$= \frac{1.6\times10^{-19}C\times2.2\times10^{6}\,m/s}{2\pi\times5\times10^{-11}m} = 1.12\times10^{-3}A$$

$$= 1.12mA$$

Example 4: The current in a wire varies with time according to the relation I = 2.0A (0.6A/s^2. (a) How many coulomb of charge pass a cross-section of the wire in the time interval between t =0 and t = 10s? (b) What constant current would transport the same charge in the same time interval?

Solution:

a)

$$q = \int_0^t idt = \int_0^{10s} [2.0A + (0.6As^{-2})t^2]dt$$
$$= \left[(2.0A)t + (0.6As^{-2})\frac{t^3}{3}\right]_0^{10s}$$
$$= (2.0A)\,(10s) + (0.6\ As^{-2})\frac{(10s)^3}{3} = 20As + 200As$$
$$= 220As = 220\ coulomb$$

b)

Constant Current i= $i = \frac{q}{t} = \frac{220C}{10s} = 22A$

OHM'S LAW

On the basis of his experimental observations Ohm (Germany) Scientist discovered a law, known as Ohm's Law, which states that the current flowing through a conductor is directly proportional to the potential difference across its ends, provided the physical conditions of the conductor remain the same.

Current ∝ potential difference or I ∝ V …….[Ohm's Law]

So, we can also write V∝I or, V= RI

Where R is the constant of proportionality, called the resistance of the given conductor.

Resistance

The property of a substance due to which it opposes the flow of current through it is called its 'electrical resistance'. For a given body it is defined as the ratio of applied potential difference to the resulting current.

Unit of Resistance. The SI unit of electrical resistance is called Ohm (denoted by symbol Ω). From Ohm's law, we have:

R = $\frac{V}{I}$ → 1 ohm = $\frac{1\ volt}{1\ ampere}$

Thus, the resistance of a conductor is said to be 1 ohm if a current of 1 ampere flows through it when the potential difference across its ends is 1 volt.

Remember:

$R = V/I$, defines Resistance R for any conductor, whether it obeys Ohm's law or not i.e. ohmic as well as non-ohmic conductors. But only when R is constant, this relation $R = V/I$ is called Ohm's Law.

What causes resistance in a conductor?
The resistance of a wire depends upon the number of collisions the electron makes with the atoms of the wire. Therefore, resistance depends on the arrangement of atoms in the material i.e. on the kind of material (copper, silver etc.). Resistance must also depend upon the length and thickness of the wire. Thus, the resistance of a wire depends upon: a) the material of the wire b) its length c)its thickness.

Electrical Resistivity or specific Resistance of Materials:

The experiment shows that the resistance of two wires of different materials, say copper and silver having the same length and the same diameter (i.e. the same cross – section) depends on the material of which the wire is made. The experiment also shows that if we take two wires of the same material and having the same diameter, their resistances will be different if they have different lengths. In fact, if the length is doubled, the resistance is also doubled, i.e. resistance R is directly propostional to the length of the wire.

$R \propto l$ ......(i)

The experiment also finds that if we take two wires of the same material having the same length, their resistance will be different if they have different areas of cross-section. In fact, if cross-sectional area A is doubled, the resistance R becomes one-half, i.e., resistance is inversely proportional to the area of the cross section of the wire

$R \propto \frac{1}{A}$ ......(ii)

Combining (i) and (ii) we have

$R \propto \frac{l}{A}$ or R=$\rho \frac{l}{A}$

Or $\rho \frac{RA}{l}$ .....(iii)

Where ρ a constant of proportionality and its value is depends on the material of the wire. The constant ρ is called the resistivity of the material of the wire.

l = 1 m and A=1m^2 then ρ = R

Thus, the resistivity of the material of a wire is the resistance of a wire of length 1 m and area of cross-section equal to 1 m^2.

Unit of ρ = ohm metre.

Note that the value of ρ is independent of the length and the area of cross-section of the wire; it depends only on the material of the wire. Thus, resistivity is a characteristic of the material.

The reciprocal of electrical resistivity ($1/\rho$) is called electrical conductivity (σ), i.e., $\sigma = 1/\rho$; σ is expressed in ohm^{-1}m^{-1} or Siemen per meter (Sm^{-1}).

Conductance:

The reciprocal of resistance is called conductance. It is denoted by G. So, G = $\frac{1}{R}$
SI unit of conductance is ohm^{-1} (Ω^{-1}) or mho or Siemen (S).

Important:

If l1 and l_2 be the initial and final lengths, and r_1 and r_2 be the initial and final radii of their cross sections, then

$$A_1 l_1 = A_2 l_2 \Rightarrow \pi r_1^2\, l_1 \Rightarrow \pi r_2^2 l_2 \Rightarrow \frac{l_1}{l_2} = \frac{r_2^2}{r_1^2} = \frac{A_2}{A_1}$$

Example 5: A wire of length L and cross-section area A has resistance R. What will be the resistance of the wire if it is stretched to twice its original length? Assume that the density and resistivity of the material do not change when the wire is stretched.

Solution: When the length is doubled, the cross-section area is halved so that volume remains same. Hence, new length and new area so the new resistance

What Cause Resistance in a Conductor?

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Electrical Resistivity or Specific Resistance of Materials

The experiment shows that the resistance of two wires of different materials, having the same length and the same diameter (i.e., the same cross-section) depends on the material of which the wire is made. The experiment also shows that if we take two wires of the same material and having the same diameter, their resistance will be different if they have different lengths. In fact, if the length is doubled, the resistance is also doubled, i.e., resistance R is directly proportional to the length of the wire.

Colour Code for Resistors

The value of resistances used in electrical circuits vary over a very wide range. A colour code is used indicate the value of resistances.

A resistor has usually four concentric rings or bands A,B,C and D of different coulours. The colours of first two bands A and B indicate the first two significant figures of the resistances in ohm, while the colour of third band C indicates the decimal multiplier. The colour of fourth ring or D (which is either silver or gold) tells the tolerance. Sometimes, only three colour band A, B and C are marked.

The colour of first two bands A and B correspond to figures 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and the colour of the third band C corresponds to multipliers $10^0, 10^1, 10^2, 10^3, 10^4, 10^5, 10^6, 10^7, 10^8, 10^9$, respectively. If the colour of the fourth band is gold, the tolerance is 5% and in case the colour is silver, the tolerance is 10% in case, there no fourth, then its tolerance is 20%.

$R = AB \times 10^c \pm D\%$

The following table gives the colour code for carbon resistance:

Letters	Colour	Figure	Multiplier	Figure	Figure
B	Black	0	10^0	Gold	5%
B	Brown	1	10^1	Silver	10%
R	Red	2	10^2	No color	20%
O	Orange	3	10^3		
Y	Yellow	4	10^4		
G	Green	5	10^5		
B	Blue	6	10^6		
V	Violet	7	10^7		
G	Grey	8	10^8		
W	White	9	10^9		

To remember the colours in correct sequence, one may remember B.B. ROY in Great Britain has Very Good Wife. The capital letters correspond to colours in the correct sequence.

Drift Velocity of free Electrons

When we join the end of the metallic wire to the battery, a potential difference is established across the terminals of the wire. An electric field is developed throughout the wire. This field accelerates the electrons but the speed of the electrons does not increase continuously because they lose energy due to collisions with the positive ions.

Thus, the electric field gives a constant average velocity to the free electrons along the length of the wire. This velocity is called "drift velocity (v_d) of electrons. The order of drift velocity is 10^{-3} m/s.

Consider a metal having n free conduction electrons per unit volume. Let v_1 be the random velocity of a conduction electron just after collision with a positive ion.

When a potential difference is applied across the metal, an electric field E is set up. The force on each electron is$e\underset{E}{\rightarrow}$. The force acts on the electron in a direction opposite to the direction of the electric field. The momentum gained by the electron.

Here is the time interval between two successive collisions of a conduction electron (also called relaxation time)

In above equation, v_d represents average drift velocity of the electrons. The negative sign just indicates that electrons drift in a direction opposite to the field E.

Current Density

We now define the concept of "current density", a vector quantity that points in the direction of the electric field. Its magnitude is defined as the current per unit cross-section area normal to the direction of electric field.

Relation between Current Density and Drift Speed
We have already derived that the current

$I=neAv_d$ ………… (current)

Hence, $j=\frac{i}{A}$ = new_d ………(current density)

Mobility: The mobility (μ) is defined as the magnitude of drift velocity per unit electric field. That is

…………. (mobility)
μ is positive for both positive and negative charges.

…………..(mobility)

If the mobile charge carrier particle has charge q, then

The conductivity, $\sigma = \frac{j}{E}$ …….(ohm's law)

Or,

Example 9: What is the drift velocity of electrons in a silver wire of length 1 m, having cross-sectional area 3.14 x 10^{-6} m^2 and carrying a current of 10A. Given, atomic weight of silver = 10^8, density of silver 10.5 x 103 kg/m^3, charge of electron 1.6 x 10^{-19} C, Avogadro's number = 6.023 x 10^{26} per kg.

Solution: Use the formula I = n.e.v_dA

First calculate n = no. of electrons per unit volume. Imagine the volume of silver to be 1m^3

Now its mass = density x volume = $10.5 \times 10^3 \frac{kg}{\mathrm{m3}}$ x1m^3 = 10.5x10^3kg

So, no of silver atoms in this will be got by multiplying the number of moles by Avogadro's number.

Now, since the valency of silver is one, we can assume each atom of silver contributes one electron. So finally

Example 10: A potential difference V is applied to a conducting wire of length l and diameter d. How are the electric field E, the drift speed v_d and the resistance R affected if (i) V is doubled (ii) l is doubled (iii) d is doubled?

Solution: we have, electric field E

Drift speed = v_d

And resistance R =

Hence, (i) When V is doubled, E is doubled, v_d is doubled and R remains same

(ii) When l is doubled, E is halved, v_d is halved and R is doubled

(iii) When d is doubled, E remains same, v_d remains same and R becomes One – fourth

DEPENDENCE OF RESISTIVITY

Over a limited temperature range (upto 100^0 C or so), the resistivity of a metal can be resistivity of a metal can be represented approximately by the linear equation

……………… (temperature dependence of resistivity)

Where ρ_0 is the resistivity at a reference temperature T_0 (often taken as 0^0C or 20^0C), ρ_T is the resistivity at temperature T which may be higher or lower than T_0

(a) For manganin

The factor α is called the temperature coefficient of resistivity.

The temperature dependence of resistance is same as that of resistivity because R$\propto$ p. hence for metals

$R_T = R_o [1+\alpha(T-T_0)]$ …………… (resistance at temperature T)

Where α is called the temperature coefficient of resistance.

THERMISTOR

A thermistor is a semiconductor device with a temperature-dependent electrical resistance. It is commonly used in medical thermometers and to sense overheating in electronic equipment. It acts on the principle that the temperature coefficient of resistivity is negative and large in magnitude for semiconducting materials.

LIMITATIONS OF OHM'S LAW

Usually is a common error to say that $\frac{V}{i}$ =R is a statement of ohm's law. This equation is simply the defining equation for resistance and applies to all conducting devices, whether they obey Ohm's law or not.

1. The essence of Ohm's law is that the value of R is independent of V, that is, $V \propto i$.
2. According to Ohm's law the graph of V versus i is a straight line.
3. Ohm's law is more general for conducting materials rather than for conducting devices.

Modern microelectronics civilization depends almost totally on devices that do not obey Ohm's law. Following graphs show the variation of i with V for some circuit devices. Figure (a) is a plot for a resistor obeying Ohm's law. The slope $\frac{i}{V}$ of the straight line is same for all values of V. this means that the resistance R = $\frac{V}{i}$ of the resistor is independent of the magnitude and polarity of the applied potential difference V.

(a) For resistor

It is a plot when the device is a semiconducting pn junction diode. Current flows through the device only when the polarity of V is positive and applied potential difference is more than 1.5 V approximately. And when current does flow, the relation between i and V is linear, it depends on the value of the applied potential difference V. Hence, Ohm's law is not obeyed.

(b) For a pn-junction diode

SUPER-CONDUCTIVITY

The phenomenon of sudden disappearance of all electrical resistivity at a low temperature is known as superconductivity.

The low temperature at which all electrical resistivity disappears suddenly is called critical temperature (T_c) its value is different for different materials.

COMBINATION OF RESISTORS IN SERIES AND PARALLEL

For any combination of resistors we can always find a single resistor that could replace the combination without changing the total current through the combination or the potential difference across the combination the resistance of a single resistor is called the equivalent resistance of the combination.

If V = potential difference between the terminals of a network
I = current at the terminals
Then equivalent resistance is $R_{eq} = \frac{V}{i}$