ACCELERATION

Acceleration is the rate of change of velocity with time. The concept of acceleration is understood in non-uniform motion. It is a vector quantity.

Average acceleration is the change in velocity per unit time over an interval of time.

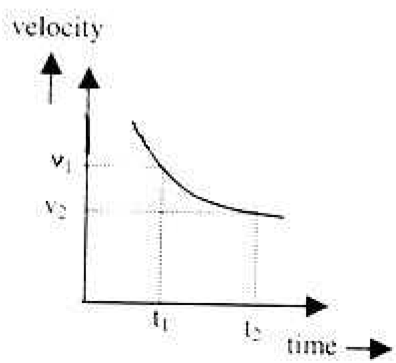
$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta\vec{t}} = \frac{\vec{v}_2 - \vec{v}_1}{\vec{t}_2 - \vec{t}_1}$$

Instantaneous acceleration is defined as

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\Rightarrow \vec{a}\ \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} + \frac{dv_z}{dt}\hat{k} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

Acceleration vector in non uniform motion



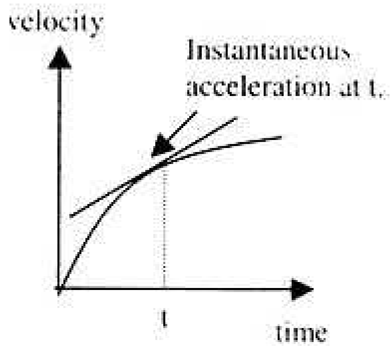
Suppose that at the instant t_1 a particle as in figure above, has velocity $\vec{v}_1$ and at t_2, velocity is $\vec{v}_2$. The average acceleration < $\vec{a}$> during the motion is defined as

$$< \vec{a} > = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}$$

Variable Acceleration

The acceleration at any instant is obtained from the average acceleration by shrinking the time interval closer zero. As Δt tends to zero average acceleration approaching a limiting value, which is the acceleration at that instant called instantaneous acceleration which is vector quantity.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$



i.e. the instantaneous acceleration is the derivative of velocity.

Hence instantaneous acceleration of a particle at any instant is the rate at which its velocity is changing at that instant. Instantaneous acceleration at any point is the slope of the curve v (t) at that point as shown in figure above.

Equations of motion

The relationship among different parameter like displacement velocity, acceleration can be derived using the concept of average acceleration and concept of average acceleration and instantaneous acceleration.

When acceleration is constant, a distinction between average acceleration and instantaneous acceleration loses its meaning, so we can write

$$\vec{a} = \frac{\vec{v} - \vec{v}_0}{t - t_0} = \frac{d\vec{v}}{dt}$$

where $\vec{v}_0$ is the velocity at t = 0 and $\vec{v}$ is the velocity at some time t

Now $\vec{a}\ t = \vec{v} - \vec{v}_0$

Hence, $\vec{v} - \vec{v}_0 = \vec{a}\ t$ .... (2)

This is the first useful equation of motion.

Similarly for displacement

$$\vec{x} = \vec{x}_0 + < \vec{v} > t \qquad .... (3)$$

in which $\vec{x}_0$ is the position of the particle at t_0 and < $\vec{v}$ > is the average velocity between t_0 and later time t. If at t_0 and t the velocity of particle is

$$< \vec{v} > = ½ (\vec{v}_0 + \vec{v})$$

$$= ½ [\vec{v}_0 + \vec{v}_0 + \vec{a}\ t]$$

$$< \vec{v} > = \vec{v}_0 + \vec{a}\ t/2 \qquad .... (4)$$

From equation (3) and (4), we get

$$\vec{x} - \vec{x}_0 = \vec{v}_0\, t + \frac{1}{2}\vec{a}\ t^2 \qquad .... (5)$$

This is the second important equation of motion.

Now from equation (2), square both side of this equation we get

$$v_2 = v_0^2 + a^2t^2 + 2v_0\, \vec{a}\ t = v_0^2 + 2\,\vec{a}\ t + [v_0 + \vec{a}\ t/2]$$

$$= v_0^2 + 2\,\vec{a}\ t < v > \text{(Use equation 4)}$$

Use equation (3), to get

$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}(\vec{x} - \vec{x}_0) \qquad .... (6)$$

This is another important equation of motion.

- **Caution:** The equation of motion derived above is possible only in uniformly accelerated motion i.e. the motion in which the acceleration is constant.